

# The existence of a minimum wavelength for photons

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The holographic property of entropy plays a key role in the thermodynamic description of gravitational field equations. It remains unclear, we argue, whether this property is necessarily interwoven with gravity itself or can be understood instead as a manifestation of physics outside gravity.

It is pointed out that if the latter is the case, so that the holographic property of entropy could be considered as completely rooted on some microscopic, non-gravitational, intrinsic property of matter, gravity could merely be understood as the necessary macroscopic manifestation of this microscopic property of matter, being simply the expression of the thermodynamic conservation of energy. A quite peculiar status of Einstein's gravity in this context (as compared to generalized metric theories of gravity) is apparent. A successful microscopic property of matter, is found to necessarily mean in particular the existence of an absolute minimum wavelength for photons, of orders of Planck length.

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In the fundamental paper [1], Jacobson showed that the Einstein equation can be viewed as an equation of state. It should then consist in a relation among thermodynamic quantities, describing the thermodynamic limit of an unknown underlying theory of microscopic spacetime degrees of freedom, i.e. it would be expression of thermodynamics of spacetime.

This being gravity a thermodynamic manifestation of spacetime has turned out to be not peculiar to Einstein's theory. The result [1], indeed, has been shown to hold true for the equations of motion of generalized theories of gravity [2, 3]. An expression/formulation of the action principle, with direct thermodynamic meaning, has also been found for any diffeomorphism invariant theory [4]. Moreover, an on-shell equipartition law has been shown to hold, expressing the energy of a gravitating body in terms of spacetime degrees of freedom living on space 2-surfaces enclosing the source [5, 6].

In [7] the attempt was made to introduce a complementary thermodynamic perspective, in which the Einstein equation could also be understood as merely what must happen in order that a certain intrinsic property of (matter) entropy be preserved. This could be intended as an effort to try to inquire further, within the line opened in [1], as to the meaning of having the equation a statistical nature.

The crucial property of matter entropy used in [7] is that expressed by the holographic principle [8, 9], in the formulation through the generalized covariant entropy bound (GCEB) [10]. This property has been suggested from combined consideration of the existence of gravitational collapse in general relativity and of the basic principles of quantum mechanics. It could seem thus circular to use it to 'derive' Einstein's equation. The proposal in [7] was, however, to consider it as a primeval, or intrinsic, property, in the sense of something which, being intrinsic to matter, is supposed to come 'before' gravity. Thus, something whose roots/justification are not in gravity. From this perspective, the problem of having an understanding of gravity seems then shifted to gain insight into the physical meaning of a non-gravitational holographic property for entropy. Aim of the note is to present two remarks upon this.

The first remark is as follows. The holographic property just mentioned is usually formulated saying that the entropy of a patch of horizon is proportional to the area,  $S = \eta A$  with  $\eta$  a constant, and this expression was indeed assumed in [1]. Its motivation in [1] was not the holographic principle, but the possible identification of  $S$  with the (finite) entanglement entropy associated to the correlations between the modes on the two sides of the horizon, not having, as such, any reference to gravity (thinking, locally, to the correlations on the two sides of a Rindler horizon in flat spacetime). This property of entropy is thus 'intrinsic' with the very same meaning as we said for [7]. A finite entanglement entropy is obtained assuming there is a fundamental cutoff length  $l_c$  in the modes of the quantum field theory. The entropy can then be expected to be  $S = \eta A = \alpha A/l_c^2$  for  $D = 4$  spacetime dimensions,<sup>1</sup> where the constants  $\eta$ ,  $l_c$  and  $\alpha$  are intended as not rooted on gravity.<sup>2</sup>

Let us consider the equilibrium process discussed in [1], at a point  $P$  of  $D = 4$  spacetime. A certain amount

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<sup>1</sup>  $S = \eta(D)A = \alpha(D)A/l_c^{D-2}$ , in general.

<sup>2</sup> The interpretation of (Rindler) horizon entropy as entanglement entropy has been recently questioned in [11]. There, the suggestion is made to consider it, instead, as the Shannon entropy coming from Heisenberg uncertainty associated to the boost generator of accelerated observer. As the two entropies, even if different in nature, are equal (and thus with a same functional dependence on area  $A$  of the horizon), and both are defined in flat spacetime (and, in this sense, both are independent of gravity), they seem to share the features essential for the discussion we are trying to carry on.

of matter energy  $dE$  goes beyond a local Rindler horizon at  $P$ , i.e. it is received by the ‘system’ consisting of the degrees of freedom beyond the horizon. The variation of entropy of the system is given by the variation of horizon entropy  $dS$ , i.e. by a variation of the area of the horizon, geometrically governed by the Raychaudhuri equation, and this gives a lensing, if the first law of thermodynamics for this process,  $dE = TdS$  ( $T$  is horizon temperature), has to be met. The strength of the lensing turns out to be that prescribed by the Einstein equation if  $\eta = 1/4G$  (in  $c = 1 = \hbar$  units), where  $G$  is the Newton constant.<sup>3</sup>

The emphasis in [1] is that this argument implies the Einstein equation can be viewed as playing the role of an equation of state. From the perspective of [7] the same argument can be read also as follows: the holographic property of entropy, i.e. the mere existence of the fundamental cutoff  $l_c$  in quantum modes, determines a lensing of null geodesics if the first law of thermodynamics, as applied to the local Rindler horizons, has to be satisfied.<sup>4</sup> That is to say, the spacetime of a world with a fundamental cutoff  $l_c$  in quantum field description is necessarily curved, if it has in it a local conservation of energy in the thermodynamic sense. Without a fundamental cutoff, horizon (entanglement) entropy would be infinite and the variations of it (if sensible), required by the thermodynamic first law, could be accounted for also without resort to lensing. In this perspective, gravity *is* the manifestation of the existence of  $l_c$ .

The second remark is similar to the first one. It consists in a reformulation of it in the context of the argument provided in [7]. This seemingly adds some indication on the concrete physical meaning of  $l_c$ .

In [7] the holographic property of entropy is intended to be expressed by the GCEB. In particular, the matter entropy content  $S_m$  of a thin, plane slice of matter at a given point of spacetime is, according to it, bounded from above by  $\Delta A/4G$ , being  $\Delta A$  the area variation of the cross-section of the (terminated) lightsheet built on it.

The presence of  $G$  indicates, of course, the gravitational origin of the bound. In [7] however, its existence and value are supposed to be rooted into physics outside gravity, and the gravitational focusing, with strength  $G$ , is supposed to be the necessary consequence of the universal validity of the bound. The strength of the focusing is set by requiring the bound is exactly attained in the most challenging cases. These are the lightsheets constructed on as-thin-as-possible plane layers of matter –i.e. they coincide with the local Rindler horizons of [1]–, for the most entropic matter. As apparent in [7], to require in these circumstances the validity of the GCEB is equivalent to require the validity of first law of thermodynamics for local Rindler horizons as in [1] (with  $dS_m$  on the lightsheet coinciding with  $\delta Q/T$  of [1]).

We try now to proceed further in the approach [7] and inspect the possible meaning/justification of the GCEB-based holographic property of entropy in a framework in which no reference is made to gravity. To this end we can make use of the relation  $s \leq \pi\lambda(\rho + p)$  (see [7] and references therein), which (we conjectured) is of pure quantum mechanical origin, and appears to be exactly attained by the most entropic systems, in particular by the photon gas. Here  $s$ ,  $\rho$  and  $p$  are the entropy and energy densities and pressure respectively, and  $\lambda$  is the quantum wavelength of matter constituents at the assigned thermodynamic conditions.<sup>5</sup>

Assuming the bits of information are carried by the constituent particles so that  $\lambda$  is the spatial scale assigned to  $\approx 1$  bit, what this relation says is that, at a given energy + pressure energy in a layer of 1-bit thickness there is a maximum number of bits allowed in the layer, and this maximum number is reached by the photon gas (the photon gas with the given  $\rho + p$ ). In a photon gas the distance between the photons is  $\approx \lambda$  so that the layer of photons can be seen as a layer of bits placed side-by-side, i.e. they are maximally packed. That is, given a surface with area  $A$  there is a maximum number of bits we can put in a layer on that surface, at the given energy + pressure energy in the layer. No limit can be envisaged for the number of bits, if the energy + pressure energy can grow unlimitedly. All this from quantum mechanics alone.

What the GCEB holographic property of entropy adds, is that on the area  $A$  an absolute maximum number of bits  $N_{abs}$  must exist, since  $S_m \leq \Delta A/4G \leq A/4G \equiv N_{abs}$ . The GCEB holographic property can then be rephrased as: Given a surface with area  $A$ , there is an absolute maximum number of bits of information we can have on a layer of 1-bit thickness on the surface (i.e. independent of energy and pressure energy in the layer). We see this amounts to require the existence of an absolute minimum for the size of a bit and, equivalently, an absolute minimum for the  $\lambda$  of the photons, of orders of the Planck length.

<sup>3</sup> In generalized theories (with  $D \geq 4$ ), the mentioned results [2, 3] show that, using as horizon entropy the Noether charge entropy  $S_c$  [12], the thermodynamic relation  $dE = TdS_c$  is equivalent to the field equations provided  $S_c = (1/4G_{eff})A$ , being  $G_{eff}$  the effective gravitational coupling of the theory and  $A$  the horizon area. Horizon entropy turns out to be, still, proportional to (actually, a quarter of) horizon area if the latter is measured in units of the effective gravitational coupling [13].

<sup>4</sup> A point of view on Jacobson’s result which seems not so different from that presented in [14]; there, within an information-theoretic perspective, the stress is, still, on gravity being derived from an intrinsic property of entropy.

<sup>5</sup> More precisely,  $\lambda$  is defined as that spatial scale  $l$  at which the momentum quantum uncertainty induced on the constituent particles –if imagined physically constrained in  $l$ – is equal to the intrinsic spread in momentum at the assigned thermodynamic conditions.

Thus:

- i) The ultimate physical content of the holographic property of entropy would be the existence of an absolute minimum wavelength  $l_m$  for photons;
- ii) The reason of the existence of  $l_m$  would not be rooted in gravity;
- iii) Gravity would be the macroscopic manifestation of the existence of  $l_m$ , and would consist in the thermodynamic conservation of energy in all circumstances, Rindler observers included.

This concludes our second remark.

A final comment. Changing the functional expression of the entropy assigned to causal horizons means to change the gravitational equations of motion implied by the thermodynamic first law. As mentioned in footnote 3, there is evidence [13] that horizon entropy in generalized theories of gravity is still given by a quarter of the area, provided the horizon area is measured in units of the effective gravitational coupling  $G_{eff}$ . The identification of horizon entropy with entanglement entropy is thus still a viable option, with cutoff length  $l_c$  (or the minimum photon wavelength  $l_m$  of the photon gas of the second remark) depending on the theory of gravity (see also [15]). The existence of a cutoff length can be suspected as the general recipe for a thermodynamic description of the equations of motion to be feasible. However, in generalized theories  $l_c$  would have a dependence on curvature, contrary to Einstein's theory, and its value will be set by combined action of non-gravitational physics and the gravitational metric theory under examination. In the spirit of present note, which aims at "explaining" gravity as manifestation of the existence of an absolute minimum length fully rooted in non-gravitational physics, we see that Einstein's theory seems to have a quite unique status. The Einstein theory (intended as the theory coming from the Einstein-Hilbert Lagrangian, in  $D = 4$  or in  $D > 4$  dimensions) appears to be the only metric theory of gravity which allows for a minimum cutoff length entirely motivated outside gravity. This point is crucial in the discussion here, and is perhaps worth appreciating in general when comparing the different approaches to gravity.

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